Math 107

Linear Growth

Many real life situations involve growth that grows at a constant rate – here is an example:

You might have read in the paper about the "Consumer Price Index" (CPI). The Consumer price index is used by the government to measure the "buying power" of consumers. They take a "basket of goods and services" that most everyone would purchase and use this to estimate inflation and how much it would cost someone to live. They start with a base year (say it is 2005) and use the prices of these items in 2005 and give this value 100. IF the CPI in say 2010 is 125, this means the cost of items in 2010 has increased 25%

Here is a table of CPIs (made up not actual)

Years since 2005	0	2	3	5	10
CPI	100	127.5	141.25	168.75	237.5

Graph this data using Excel:

What do you notice? This graph is exactly a line. Note the average rate of growth between any two points is exactly the same (we will now call this the rate of growth). You probably remember some ideas about linear growth from earlier math classes

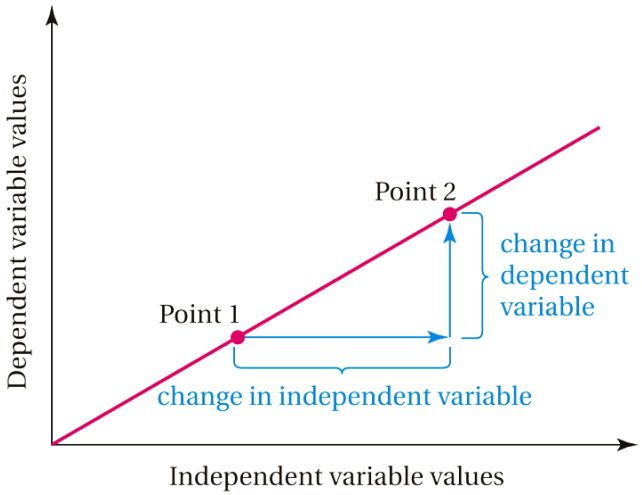
Linear Growth

Linear growth occurs when a quantity grows by the same **absolute amount** each time period.

- The rate of growth of a linear model is the slope of the line.
- The greater the rate of change the steeper then line
- Calculate the rate of change by finding the slope of the line between any two points

$$slope = \frac{change\ in\ dependent\ variable}{change\ in\ independent\ variable}$$

- A negative slope represents a decrease in population or amount



Example: Suppose you don't trust banks and instead decide to hide your money in a cookie jar. You start with a $100 that your aunt gave you for a Bat Mitzvah present and you decide to deposit $25 a month. We can create a table of values thinking of the month as the independent variable and the value as the dependent value.

Month	Value
0	$100
1	$125
2	
3	
4	

And then create a quick graph.

What is the growth rate? What is the initial amount? These two items can help us create the typical equation of the line.

$$y = Growth\ rate\ \times x + Initial\ Value$$

In our case $v = 25 \times m + 100$ where v represents value and m represents months.

We can also write this equation using a recursive formula" since the amount always changes the same – we can write

New amount = old amount + growth rate

In our case $a_n = 25 + a_{n-1}, a_1 = 100$

Example: Suppose you have hidden $250 in a cookie jar and then take $12.50 a week to go to the movies. What is the growth rate? Is it constant? What is the initial amount? What is an equation that models this situation? How long before the cookie jar is empty?

Example: Suppose your car's 20 gallon gas tank is full when you begin a road trip and that you are using gas at a constant rate. After travelling for two hours, your fuel gauge reads three quarters full. What is the rate at which you are using gas (include units!)? What is a linear equation that models this situation?

In the real world, it would be next to impossible to have the gas being used at an exact constant rate although it might be close to an exact constant rate. In this case the data might be *approximately linear*.

Example: Gasoline consumption in the US has been increasing steadily. Consumption data from 1992 to 2004 is shown.

Year	92	93	94	95	96	97	98	99	00	01	02	03	04
Consumption (billions of gallons)	110	111	113	116	118	119	123	125	126	128	131	133	136

Is this data exactly linear? How can you tell? Is it approximately linear? How can you tell?

Example: We could use statistical techniques to find a line of best fit (called a regression line) – this is easy to do in Excel and I will show you how

Example: You might notice we can do pretty well by choosing two sets of data points and using those points to find an equation of the line. For example if you chose 1993, 111 and 2003, 133.